Understanding taxi travel patterns

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**Highlights**

- Holistic analysis of taxi travel patterns including occupied and unoccupied trips.
- Trip displacement of short trips follows power-law distribution.
- Trip displacement of long trips follows exponential distribution.
- With increased characteristic travel distances, taxis can have dual high-probability locations.

**Abstract**

Taxis play important roles in modern urban transportation systems, especially in mega cities. While providing necessary amenities, taxis also significantly contribute to traffic congestion, urban energy consumption, and air pollution. Understanding the travel patterns of taxis is thus important for addressing many urban sustainability challenges. Previous research has primarily focused on examining the statistical properties of passenger trips, which include only taxi trips occupied with passengers. However, unoccupied trips are also important for urban sustainability issues because they represent potential opportunities to improve the efficiency of the transportation system. Therefore, we need to understand the travel patterns of taxis as an integrated system, instead of focusing only on the occupied trips. In this study we examine GPS trajectory data of 11,880 taxis in Beijing, China for a period of three weeks. Our results show that taxi travel patterns share similar traits with travel patterns of individuals but also exhibit differences. Trip displacement distribution of taxi travels is statistically greater than the exponential distribution and smaller than the truncated power-law distribution. The distribution of short trips (less than 30 miles) can be best fitted with power-law while long trips follow exponential decay. We use radius of gyration to characterize individual taxi’s travel distance and find that it does not follow a truncated power-law as observed in previous research.

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1. Introduction

Understanding the dynamics of human mobility is critical to many fields such as epidemics [1,2], transportation [3,4], urban planning [5], and genetics [6]. Recent development of information and communications technology (ICT) has significantly improved our ability to collect, store, and analyze large-scale datasets, which enables studying human mobility at a wide range of spatial and temporal scales [7]. Analyzing statistical properties of bank notes circulation as a proxy of human movement, Brockmann et al. [8] observed that human travel distance exhibits a power law distribution and human travel trajectories may be approximated as Lévy flights (heavy tailed random walk) [8]. This observation was confirmed by Rhee et al. [9] using Global Positioning System (GPS) traces collected from volunteers, showing non-negligible probability of high displacement trips and long pause-time between trips [9]. Despite the randomness indicated by Lévy flight models, using cell phone traces, González et al. [10] discovered that strong spatial and temporal regularities exist for human trajectories—individuals have a tendency to frequently revisit a few locations [10]. To explain the observed scaling properties and regularities, Song et al. [11] proposed a combination of two mechanisms, exploration and preferential return, for modeling human mobility [11]. It is worth noting that data used in these studies include trajectories generated from different transportation modes (walking, driving, public transportation, and flying etc.). Yan et al. [12] argued that transportation modes affect aggregated travel patterns and displacements from single mode transportation should follow an exponential distribution instead of power law [12].

Vehicular travel is one important transportation mode that has implications to many societal challenges [13]. Due to the privacy concerns related to private vehicle travel data, most studies on examining vehicle travel patterns so far are based on taxi trajectories captured by GPS. Despite having different travel needs compared to private vehicles, taxis can be viewed as floating sensors in a city that provide invaluable information on mobility dynamics, traffic conditions, and epidemic spreading risks. In addition, compared to other datasets used in studying human mobility dynamics such as the movement of bank notes [8], smart card data [14], and mobile phone records [10], GPS traces of taxis usually provide more detailed information with finer spatial and temporal resolutions and larger sampling size.

Previous studies on taxi travel patterns have been focused on extracting passenger trips to understand passenger travel patterns [3,15–17]. Peng et al. [16] characterized taxi passenger trips for different travel needs [16]. Jiang et al. [3] verified the scaling properties of taxi passenger-trip length and suggested that such property is attributed to the underlying street topology [3]. Liang et al. [17] found that passenger trip displacements of taxis can be better approximated with exponential distributions instead of power law, possibly due to high costs associated with long taxi trips [17]. However, passenger trips are not representative of how taxis travel. In many cities, taxis always cruise around between passenger trips [18,19]. In some countries such as China, taxi drivers possess the vehicle throughout the day, sharing similar commuting patterns with private vehicles (i.e., returning home after work and then driving out for business the next day). To date, limited attentions have been paid to the patterns of taxi travels as a whole.

Understanding taxi travel patterns can also contribute to addressing several urban sustainability challenges. In particular, taxis are important components of the public transportation sector [13], providing flexible and convenient mobility solutions for urban residents. Because they are on the road all the time, taxis also contribute significantly to traffic congestion, energy consumption, and air pollution in the cities [20]. In addition, taxis in many countries are likely to be early adopters of emerging technologies (e.g., electric vehicles, connected and autonomous vehicles) and business models (e.g., Uber [21,22]). Therefore, it is important to understand the travel patterns of taxis, taking into account the taxi travels both with and without passengers. This research aims to fill this gap by examining taxi trajectory data holistically to understand how taxi travel patterns are similar or different from those of individuals and private vehicles. Results of this research can help guide traffic modeling, transportation planning, urban planning, and infrastructure development.

2. Methods

This study examines vehicle trajectory data of taxis in Beijing to study taxi travel patterns. The data are analyzed from the perspective of characteristic travel length (radius of gyration), distribution of trip distance, and spatial density distribution in each taxi's intrinsic reference frame.

2.1. Data

The dataset used in this study contains trajectory data of 11,880 taxis in Beijing (18% of the fleet), China over a period of three weeks (March 2–25, 2009). It includes a total of 255 million data points representing $3.4 \times 10^7$ miles of travel and over two million trips. Each data point contains a timestamp up to seconds of when the data was recorded, vehicle ID, vehicle
location at the recording time (longitude and latitude), and vehicle speed. We define a “trip” as vehicle traveling between two stop events at which the vehicle completely stops or idles (with speed less than 1 mph) for at least five minutes. Setting a minimum “dwell time” threshold is commonly used for trip separation in processing trajectory data passively collected by GPS devices [23]. The 5-min threshold is selected to reflect the long signal light waiting time in Beijing and reduce false positive trip separations when vehicles stop at traffic lights or idle in traffic. Although trip start/end time and locations need to be inferred from the data, substantial agreement exist between GPS-derived travel patterns and self-reported ones [24]. Using a different threshold (e.g. 3 min) changes the parameters slightly but does not change the conclusions of this study.

We also identify a home location for each vehicle as the location where it parks consistently for the greatest amount of time.

2.2. Background on Beijing taxis

Taxis in Beijing do not work for dispatch centers. Instead, they mainly provide hail services, which mean that taxis cruise along the streets and look for clients who signal their needs for taxis. The vehicles are owned by the taxi companies, along with vehicle plates and taxi licenses. Taxi drivers pay a fixed monthly fee to the taxi company for the right to operate. Each driver has the same vehicle during his or her contract time with the taxi companies and is responsible for any associated costs (e.g. fuel, maintenance, and upgrade). The driver possesses the vehicle 24/7 and normally parks it at where he/she lives when off work. These properties make Beijing taxis share some characteristics with private vehicles (e.g. parking at home at night, routine trips leaving and returning home).

2.3. Radius of gyration ($r_g$)

González et al. [10] suggested that human travel trajectories can be characterized by a time-independent characteristic travel distance—radius of gyration ($r_g$) [10]. Although the method was developed using individual travel data captured through mobile phone traces which have lower resolution compared to trajectory data collected by GPS, the concepts can be generalized. We modify this approach to analyze taxi travel by taking into account the time a vehicle spends at each location. Because data sampling intervals are not the same for all vehicles in our dataset, using the time-weighted $r_g$ can reduce the bias due to data sampling frequency variances. In particular, the $r_g$ of vehicle $i$ is calculated using Eq. (1). By using the time (in seconds) a vehicle spent at a particular location as the weight, it is equivalent to as if the GPS device was collecting data at every second. Therefore, although Eq. (1) includes a term $t$, the unit of $r_g$ calculated is still miles.

$$r_{g(i)} = \sqrt{\frac{1}{n_i} \sum_{j=1}^{n_i} t_{ij} (r_{ij} - r_{c(i)})^2} \tag{1}$$

where $r_{g(i)}$ represents the radius of gyration for vehicle $i$, $n_i$ is the total number of recorded locations for vehicle $i$, $j$ is the $j$th reported location; $t_{ij}$ is the time (in seconds) vehicle $i$ spent at the $j$th location; $r_{ij}$ is the position vector recorded for vehicle $i$ at the $j$th location; and $r_{c(i)} = \frac{1}{n_i} \sum_{j=1}^{n_i} t_{ij} r_{ij}$ is the “center of mass” of the trajectories of vehicle $i$, representing the mean location of vehicle $i$. We use the Manhattan distance in this study to provide better estimates for the actual travel distance between two locations in a road network.

2.4. Trajectory transformation

To compare the trajectories of taxis with different spatial coverage and characteristic travel distances, taxi trajectories can be rescaled and re-centered based on each individual vehicle’s intrinsic reference frame and $r_g$. Using its center of mass as the origin, each individual vehicle’s intrinsic reference frame is rotated with the moment of inertia and then rescaled with the standard deviation of the locations on each axis [10]. Similar to the calculation of $r_g$, we include the time spent at each location as weights. The transformation process is described in Eqs. (2)–(7) with more details in Ref. [10]:

$$I = \begin{pmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{pmatrix} \tag{2}$$

$$I_{xx} = \sum_{j=1}^{n_i} t_{ij} y_{ij}^2 \tag{3}$$

$$I_{yy} = \sum_{j=1}^{n_i} t_{ij} x_{ij}^2 \tag{4}$$

$$I_{xy} = I_{yx} = \sum_{j=1}^{n_i} t_{ij} x_{ij} y_{ij} \tag{5}$$
Fig. 1. Longitude and latitude over time for a typical taxi. Longitude is plotted in blue using left-hand-side y-axis, while latitude is plotted in red using right-hand-side y-axis. Green dotted line shows the midnight of a day. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

\[
\cos(\theta_i) = -I_{xy} \left( \frac{1}{2} I_{xx} - \frac{1}{2} I_{yy} + \frac{1}{2} \mu \right)^{-1} \frac{1}{\sqrt{1 + \frac{I_{x}^2}{\left( \frac{1}{2} I_{xx} - \frac{1}{2} I_{yy} + \frac{1}{2} \mu \right)^2}}} \tag{6}
\]

with \[\mu = \sqrt{4I_{xy}I_{yx} + I_{xx}^2 - 2I_{xx}I_{yy} + I_{yy}^2} \tag{7}\]

where \( I \) is the tensor of inertia; \( t_{ij} \) is the time (in seconds) vehicle \( i \) spent at the \( j \)th location; \( x_{ij} \) is the longitude recorded for vehicle \( i \) at the \( j \)th location; \( y_{ij} \) is the latitude recorded for vehicle \( i \) at the \( j \)th location; and \( \theta_i \) is the angle of rotation to transform the trajectories of vehicle \( i \) to its intrinsic reference frame.

To ensure comparativeness and consistency, the most frequently visited location is ensured to be on the positive x-axis by imposing an additional 180° rotation for vehicles with the most frequently visited locations lying in \( x_{ij} < 0 \).

To compare trajectories at different spatial scales, the rotated trajectories \((x', y')\) are then rescaled with the standard deviation of the locations as \((x'/\sigma_x, y'/\sigma_y)\) using Eqs. (8) and (9).

\[
\sigma_{x(i)} = \sqrt{\frac{1}{n_i} \sum_{j=1}^{n_i} t_{ij}(x_{ij} - x_{c(i)})^2} \tag{8}
\]
\[
\sigma_{y(i)} = \sqrt{\frac{1}{n_i} \sum_{j=1}^{n_i} t_{ij}(y_{ij} - y_{c(i)})^2} \tag{9}
\]

where \( \sigma_{x(i)} \) is the standard deviation on x-axis for vehicle \( i \); \( \sigma_{y(i)} \) is the standard deviation on y-axis for vehicle \( i \); \( x_{c(i)} = \frac{1}{n_i} \sum_{j=1}^{n_i} t_{ij}x_{ij} \) is the x-axis value of the “center of mass” of the trajectories of vehicle \( i \); and \( y_{c(i)} = \frac{1}{n_i} \sum_{j=1}^{n_i} t_{ij}y_{ij} \) is the y-axis value of the “center of mass” of the trajectories of vehicle \( i \).

3. Results and discussion

3.1. Characteristic travel distance and mass center

Strong spatiotemporal regularities exist for taxi trajectories. On a daily basis, taxi drivers leave home in the morning for business and return home at late night. This routine could be easily identified from the segments of flat lines in Fig. 1 showing the periods of time when this example taxi is parked at its driver’s home (taxi drivers in Beijing normally take the vehicle home after work, please refer to the Method section for more background information about the taxi fleet in Beijing).

Although taxis spent a significant portion of time parked at the drivers’ home location, their centers of mass (mean locations, \( r_{c(i)} \)) are highly skewed towards the city center. While taxi drivers live in the city and its vicinity (Fig. 2(a)), the mass centers of taxi trajectories are highly concentrated in the inner city (Fig. 2(b)). This could be explained by the
Fig. 2. (a) Spatial distribution of taxis home location; (b) spatial distribution of taxis centers of mass (mean location); (c) distribution of distance from home location to the center of mass following an exponential decay; and (d) probability distribution of radius of gyration \( r_g \) with \( r_0 = 1 \) mile, \( \beta = 1.5 \), \( \kappa = 12 \).

The radius of gyration \( r_g \) describes the spatial coverage of a taxi, indicating whether the trajectory of the taxi is concentrated in a small area or more spread out. The distribution of \( r_g \) for all taxis does not follow a truncated power-law distribution as observed in previous studies \([10,26]\), possibly due to the fact that taxi travels are the result of a combination of driver’s commuting needs, passenger’s travel needs, and driver’s cruise between passenger trips. Compared to the truncated power-law fit, the probability of taxis to have small \( r_g \) is significantly lower. Only 1.5% of taxis in this dataset have \( r_g \) that is less than or equal to 3 miles. Activities of these taxis mainly concentrate in the city center. The home locations of these taxis also tend to be close to the city center as well. Considering Tiananmen Square as the center of Beijing, the average distance from home to the center of the city is only 12.9 miles for these taxis. Approximately 93.5% of the taxis have \( r_g \) between 3 and 20 miles. The probability of a taxi to have \( r_g \) that is within this range is much higher than the probability obtained from the truncated power-law distribution. About 0.1% of the taxis have \( r_g \) greater than 100 miles mostly due to a few extremely long trips that are probably prearranged business travel rentals or drivers’ personal leisure travels.

3.2. Distribution of trip distances

The distribution of trip distances is another important measure for travel patterns. Two models exist to describe trip distance distribution: the truncated power-law distribution and the exponential distribution. González et al. \([10]\) found that truncated power-law in the form of \( P(\Delta r) = (\Delta r + \Delta r_0)^{-\beta} \exp(-\Delta r / \kappa) \) can well approximate individual trip displacement for individual people using cell phone traces, with \( \beta = 1.75 \pm 0.15 \), \( \Delta r_0 = 1.5 \) km, and \( \kappa = 80–400 \) km \([10]\). The scaling of a truncated power law means that the probability of taking long trips is non-negligible. Liang et al. \([17]\) argued that the displacement of taxi passenger trips follows an exponential decay with exponent \( \lambda = 0.1689–0.2329 \), indicating that the likelihood of large displacement trip decreases exponentially and is negligible \([17]\). We use both models to fit the trip displacements observed in this study. Our results show that the tail of our data falls between these two fitted models (Fig. 3(a)). Model parameters estimated in this research are in the similar range with aforementioned studies. This indicates that the probability of taxis taking long distance trips \( (d \geq 30 \) miles) is not as significant as what a power-law distribution predicts, but is greater than what an exponential decay implies. We verified that the distribution of the empirical data is statistically greater than the fitted exponential distribution and smaller than the fitted truncated power-law distribution (p-value = 0.665, details provided in the Supplemental information, Appendix A). Based on the observed differences, we further fitted the probability distribution of short trips \( (d < 30 \) miles) and long trips \( (d \geq 30 \) miles) separately and
Fig. 3. The probability density function $p(d)$ of travel distance $d$ fitted with different models. (a) The entire curve is fitted with two previously identified models: (1) exponential distribution with $\alpha = 0.06$, $\lambda = 0.068$; and (2) truncated power-law distribution with $d_0 = 6.9$ miles, $\beta = 1.254$, and $\kappa = 107.6$ miles. (b) Short trips ($d < 30$ miles) and long trips ($d \geq 30$ miles) are fitted separately: the short trips can be best fitted with a power-law distribution with $\alpha = 0.06$, $\beta = 0.061$; and the long trips can be best fitted with an exponential distribution with $\alpha = 0.02$, $\lambda = 0.038$.

Fig. 4. Probability density function of finding a taxi in its intrinsic reference frame, for taxis whose $r_g$ is greater than 4 miles but less than 5 miles.

found that the distribution of short trips follows power law while the distribution of long trips is exponential (Fig. 3(b), $p$-value $= 0.309$, details provided in the Supplemental information, Appendix A).

3.3. Spatial density distribution

The normalized spatial density distribution represents the probability of finding a taxi at a location in its intrinsic reference frame (Fig. 4). When $r_g$ is less than 10 miles, similar spatial distribution is observed with those obtained using individual mobile phone traces and private vehicle trajectories [10,27]. It is more likely for taxis to travel to places that are closer to their centers of mass and less likely to visit places that are farther away. However, when $r_g$ increases, slightly different patterns emerge (Fig. 5(a)). Instead of having one location with high probability (Fig. 5(b)), vehicles with higher $r_g$ have two separate high-probability locations (Fig. 5(c)). To identify what has caused the multiple high-probability locations, we separated the trajectories of occupied trips and unoccupied trips and plotted the spatial density distribution separately. The emergence of two high-probability locations is only observed in unoccupied trips but not in occupied trips (Fig. 6), which further confirmed the difference between trajectories of these two types of trips and the importance of including unoccupied trips to study taxi travel patterns.

4. Conclusions

In this paper, we examine the travel patterns of taxis from a holistic vehicle travel perspective, which considers not only the trips to satisfy passenger needs, but also the movement of taxis to satisfy the travel needs of the drivers (e.g. leaving from home for business and returning to home after work, cruising along streets for customers, personal travels). Our results show that taxi travel patterns share similar traits with travel patterns of individuals, but also exhibit specific differences. Trip displacement distribution of taxi travels is statistically greater than the exponential distribution and smaller than the truncated power-law distribution. The distribution of short trips (less than 30 miles) can be best fitted with power-law
Fig. 5. (a) Probability density function of finding a taxi in its intrinsic reference frame; (b) probability density function projected onto one dimension for vehicles with $r_g$ between 4 and 5 miles; and (c) probability density function projected onto one dimension for vehicles with $r_g$ between 12 and 13 miles.

Fig. 6. Probability density function of finding a taxi in its intrinsic reference frame for selected $r_g$ range for (a) trajectories of unoccupied trips and (b) trajectories of occupied trips.

while long trips (equal to or greater than 30 miles) follow exponential decay. Characteristic travel distance does not follow a truncated power-law as observed in previous studies which only examined passenger trips. Spatial and temporal regularities exist in taxi travels when the trajectories are transformed into each vehicle’s intrinsic reference frame. However, with increasing spatial coverage, taxis with greater characteristic travel distance appear to have dual high-probability locations.
These results show that characteristics of passenger taxi trips cannot be used to represent the vehicular travels of taxis. It is important to include non-occupied trips to fully understand the travel patterns of taxis from the vehicular travel perspective. Results from this study can inform taxi travel modeling (e.g., trip generation, agent-based modeling) and improve transportation system analysis (e.g., electric taxi adoption and charging behavior, charging infrastructure siting, taxi ride sharing). It is worth noting that urban form, infrastructure and taxi regulations can impact taxi travels in different cities. While we expect that the importance of holistically analyzing taxi trips persists, further research using data from multiple cities is needed to examine to what extent the specific results observed in this study can be generalized. The methods developed in this study can be applied to analyze data from other cities.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physa.2016.03.047.

References


